

SETS

✓ **Set** : A set is well defined collections of objects.

- (i) Objects, elements and members of set are synonyms terms.
- (ii) Sets are usually denoted by capital letters A, B, C, X, Y, Z etc.
- (iii) The elements of a set are represented by small letters a, b, c, x, y, z etc.

✓ **Roaster or tabular form** : Elements are listed, separated by commas and enclosed within curly brackets { }
Example : {a, e, i, o, u} set of vowels.

✓ **Set builder form** : All elements possess a single common property. Example : {x : x is a vowel in English alphabet}

✓ **Cardinal number** : Number of elements of a Set A is called cardinal number and denoted by $n(A)$.

✓ **Empty Set** : A set which does not contain any element is called the **empty set** or the **null set** or the **void set**.

✓ **Finite Set** : A set which is empty or consists of a definite number of elements is called **finite set**.

✓ **Infinite Set** : A set which is not empty & consists of a indefinite number of elements is called **infinite set**.

✓ **Equal Set** : Two sets A and B are said to be **equal** if they have exactly the same elements and we write $A = B$.
Otherwise, the sets are said to be **unequal** and we write $A \neq B$

✓ **Subset** : A set A is said to be a **subset** of a set B if every element of A is also an element of B.

$$A \subset B \text{ if } a \in A \Rightarrow a \in B$$

✓ **Proper subset** : If $A \subset B$ and $A \neq B$, then A is called a **proper subset** of B and B is called **superset** of A.

✓ **Singleton set** : If a set A has only one element, we call it **singleton set**.

📍 **Note** : Subsets of set of real numbers $N \subset Z \subset Q, Q \subset R, N \not\subset T$ T = Irrational numbers

✓ **Intervals as subsets of R** :
 $(a, b) = \{x : a < x < b\}$ is an open interval, does not contain end points a & b.
 $[a, b] = \{x : a \leq x \leq b\}$ is an closed interval, contain end points also.
 $[a, b) = \{x : a \leq x < b\}$ is an open interval from a to b, including a but excluding b.
 $(a, b] = \{x : a < x \leq b\}$ is an open interval from a to b, including b but excluding a.

✓ **Length of any interval** : The number (b-a) is called the length of any of the intervals (a, b), [a, b], (a, b) or (a, b].

✓ **Power Set** : The collection of all subsets of a set A is called the **power set** of A. denoted by $P(A)$.

✓ **Universal Set** : A set that contains all sets in a given context is called **Universal Set** denoted by U.

✓ **Union of Sets** : The union of A and B is the set which consists of all the elements of B, the common elements being taken only once. The symbol 'U' is used to denote the union.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

✓ **Some Properties of the operation of union**

- (i) $A \cup B = B \cup A$ (Commutative law)
- (ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)
- (iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of U)
- (iv) $A \cup A = A$ (Idempotent law)
- (v) $U \cup A = U$ (Law of U)

✓ **Intension of Sets** : The intension of A and B is the set of all the elements which are common to both A and B. The symbol ' $\cap$ ' is used to denote the intension.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

✓ **Some Properties of the operation of intension**

- (i) $A \cap B = B \cap A$ (Commutative law)
- (ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)
- (iii) $\phi \cap A = \phi, U \cap A = A$ (Law of ϕ and U)
- (iv) $A \cap A = A$ (Idempotent law)
- (v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i.e. $\cap$ distributes over $\cup$.

✓ **Difference of Sets** : The difference of the two sets A and B in this order is the set of elements which belong to A but not to B.

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

📍 **Note** : $A - B \neq B - A$

✓ **Complement of a Set** : Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. denoted by A'

$$A' = \{x : x \in U \text{ and } x \notin A\} \text{ obviously } A' = U - A$$

✓ **Some properties of Complement Sets**

- 1. Complement laws : (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$
- 2. De Morgan's law : (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
- 3. Law of double complementation : $(A')' = A$
- 4. Laws of empty set and universal set : $\phi' = U$ and $U' = \phi$

✓ **Practical Problems on Union and intension of two sets** :

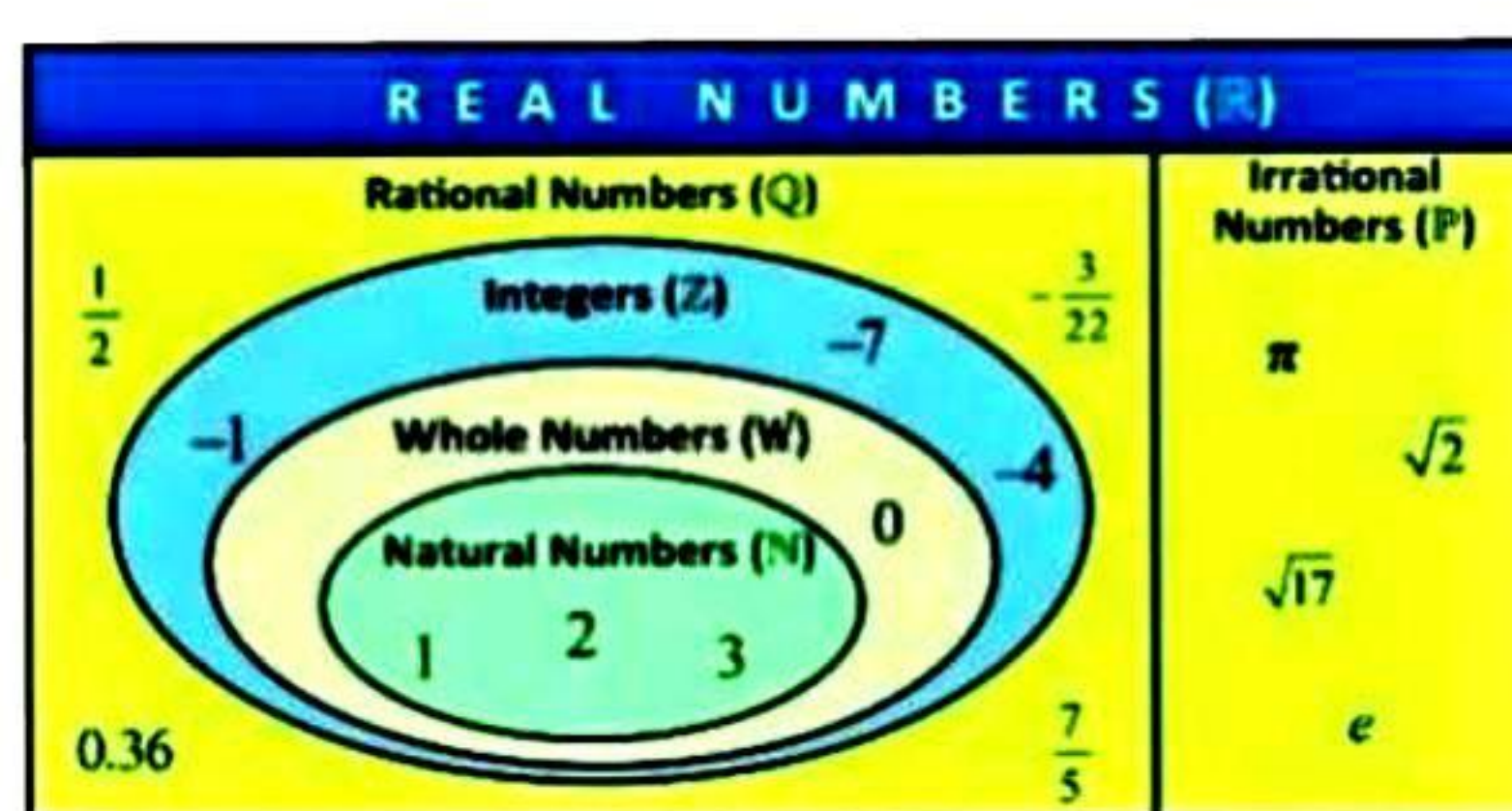
- (i) $n(A \cup B) = n(A) + n(B)$
- (ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- (iii) If A, B and C are finite sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

📍 **Note** : If A is a subset of the universal set U, then its complement A' is also a subset of U.

📍 **Important Note** :

=	equal to	<	less than
≠	Not equal to	≤	less than equal to
⊂	subset	>	greater than
⊄	not a subset	≥	greater than equal to
⇒	implies	⊃	Superset
⇔	if and only if	⊄	Not superset
∈	belongs to OR contains in	∪	Union
∀	for all	∩	Intension
:	such that		



- N : the set of the all natural numbers.
- Z : the set of the all integers.
- Q : the set of the all rational numbers.
- R : the set of the all real numbers.
- Z^+ : the set of the all positive integers.
- Q^+ : the set of the all positive rational numbers.
- R^+ : the set of the all positive real numbers.